EXPERIMENT NO. 2

EXPERIMENTAL STUDY OF
THE STATIC RESONSE OF A BEAM

Submitted by:

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1. INTRODUCTION

The purpose of this experiment is:

1. To determine the tensile modulus, i.e., Young's modulus, of a simple beam.
2. To determine the shear modulus of a simple beam.
3. To compare the measurement capabilities of the electronic and analog displacement indicators.

These aims were achieved by performing measurements and analysis on a laboratory structure of the type shown in Figure 1.

The beam is a frequently encountered structural element that is capable of withstanding external loads through the resistance of axial extension and transverse bending. Consequently, the stress and strain analysis of beams constitutes a fundamental understanding of the mechanics involved in engineering components. Often characterized by their cross-sectional shape, length, and material, beams can be supported in a variety of ways, namely simple supports such as pins and rollers and fixed connections such as when attached to a wall. Each characteristic of the beam dictates its applicability to the design situation: cross sectional shape plays an important role in the beam's ability to resist bending and torsion; length gives insight into how much the beam will deflect; material depicts the beam's ability to withstand loads before breaking. Intuitively, these characteristics work together in designing a structurally sound yet efficient system.

In the present study, a cantilever beam arrangement, shown in Figure 2, is used to measure the deflection of a simple beam due to applied transverse loads and applied torques. Knowing the relative displacements along the beam, and by assuming that plane sections remain plane, it is possible to determine the tensile modulus through the use of the Euler-Bernoulli theory of slender beams. The Euler-Bernoulli equation for the bending of slender, isotropic, homogeneous beams of constant cross-section under an applied transverse load $q(x)$ is

$$EI\frac{d^4w(x)}{dx^4} = q(x) \tag{1}$$

where E is the tensile modulus, I is the second area moment of inertia of the cross section, x is the distance along the neutral axis of the beam, and $w(x)$ is the deflection of the neutral axis of the beam. The second area moment of inertia can be determined by

$$I = \frac{1}{12}bh^3 \tag{2}$$

where b is the width of the beam and h is the height of the beam. The validity is based on the additional conditions that:

1. The beam is subjected to pure bending only.
2. The beam material is isentropic, homogenous, and linearly elastic.
3. The beam is initially straight with a cross section that is constant along the length of the beam.
4. The beam has an axis of symmetry in the plane of bending.

For a cantilever beam, the Euler-Bernoulli equation simplifies to

$$w(x) = \frac{Px^2(3L-x)}{6EI} \tag{3}$$

where P is the applied load and L is the length of the beam from the support to the location of the applied load. The maximum deflection of the beam follows as

$$w_{max} = \frac{PL^3}{3EI}. \tag{4}$$

Measuring the rotation of the beam due to a torsional load yields the shear modulus. While the assumption that plane sections remain plane is valid only for beams of circular cross section, an approximate solution can be found for the case of a rectangular cross section. For a beam of uniform cross section along its length, the angle of twist θ is

$$\theta = \frac{TL}{JG} \tag{5}$$

where T is the applied torque, L is the length of the beam from the support to the location of the applied load, J is the torsional stiffness, and G is the shear modulus of the material. The torsional stiffness, or polar moment of inertia, for a beam of rectangular cross section can be computed from

$$J = \frac{bh}{3}(b^2 + h^2). \tag{6}$$

2. APPARATUS AND TECHNIQUES

2.1 The Beam

The simple aluminum alloy beam, shown in Figure 1, is of length 18 inches, has a rectangular cross section with dimensions 1.509 ± 0.0005 by 0.255 ± 0.0005 inches, and has a weight of 0.668 ± 0.0005 pounds. Attached on each side of the beam are two electrical resistance strain gages.

2.2 The Loading Fixture and Weights

The loading fixture and one of the weights is shown in Figure 3. The loading fixture can be slid onto the beam and tightened at a location along the beam. The weights are of an aluminum bronze rod stock and are 5 inches in diameter. The set of weights used along with the loading fixture are 2.128 ± 0.0005, 3.212 ± 0.0005, and 5.134 ± 0.0005 pounds. Each has a hook for hanging from the load fixture.

2.3 The Frame

The loading frame, shown in Figure 1 with the beam, loading fixture, and weight, is built according to the design by Durelli, et al. (1965). A vertical beam support is placed 2.5 ± 0.0005 inches from the slotted side of the frame, at a height where the slot holds the beam, along with a clamp, thus approximating a cantilever support. A series of holes are spaced in two inch intervals along the top of the frame to allow the displacement indicators to measure the beam's deflection.

2.4 Other Items of Equipment

The mechanical dial displacement indicators, manufactured by the Chicago Dial indicator Company and of the model # 2-C100 1000, measure graduations with an accuracy of 1/1000 of an inch. The Mitutoyu Model 575-123 electric displacement indicators read in increments of 1/2000 of an inch.

3. RESULTS AND DISCUSSION

3.1 Tensile Modulus Measurements

Measurements to determine the tensile modulus, or Young's modulus, were performed by attaching various masses to the cantilever beam at a distance of 13.25 inches from the support and measuring the displacement of the beam in the direction of bending along the length of the beam at each of the holes in the top of the frame, as shown in Figure 3. Results for the displacements are listed in Table 1. The data was subsequently plotted and a cubic trend line was found, shown in Figures 5-7. Upon computing the beam's area moment of inertia to be 0.00209 inches^4, the empirical formula for the beam's displacement as a function of length along the beam is compared to that of Equation (3), allowing the tensile modulus of the beam to be found. The tensile modulus found for each mass is listed in Table 2. Using Equation (4), the determined tensile modulus is used to compare the maximum deflection of the beam with the measured results. These data are also included in Table 2.

Table 1 Measurements of displacement along the beam for various masses.

Mass ($lbs.$)	**$3\ in.$**	**$5\ in.$**	**$7\ in.$**	**$9\ in.$**	**$11\ in.$**	**$13\ in.$**
2.128	$0.0085\ in.$	$0.0205\ in.$	$0.0350\ in.$	$0.0515\ in.$	$0.0705\ in.$	$0.0880\ in.$
3.212	$0.0140\ in.$	$0.0310\ in.$	$0.0450\ in.$	$0.0800\ in.$	$0.1085\ in.$	$0.1385\ in.$
5.134	$0.0220\ in.$	$0.0505\ in.$	$0.0880\ in.$	$0.1300\ in.$	$0.1770\ in.$	$0.2265\ in.$

Table 2 Tensile moduli for various masses and corresponding theoretical maximum deflection

Mass ($lbs.$)	**Tensile Modulus, E ($lb/in.^2$)**	**w_{max} ($in.$)**
2.128	8.50×10^6	0.0879
3.212	8.50×10^6	0.1326
5.134	8.50×10^6	0.212

The measurements do show an increasing pattern as the distance from the support to the location of measurement is furthered as well as when the weight of the hanging mass is increased. The maximum deflection calculations are fairly consistent with the maximum deflection data, only becoming less accurate as the weight of the mass is increased. This inaccuracy may be attributed to general fatigue in the beam after several cycles of bending.

3.2 Shear Modulus Measurements

The shear modulus of the beam was estimated by attaching various masses to the cantilever beam at a distance of 13.25 inches from the support off-center from the middle of the beam, i.e., the beam's theoretical centroid, by a distance of 0.927 inches width-wise, as shown in Figure 4. Listed in Table 3 are the measured differences in displacements in the direction of bending taken at a distance of 0.369 inches from the middle of the beam width-wise at the location of the hanging mass. Figure 8 shows the plotted data for the relative transverse displacements. Knowing the beam's torsional stiffness to be 0.300 inches^4, the empirical formula for the beam's rotation as a function of length is compared to that of Equation (3), allowing the shear modulus of the beam to be found. The shear modulus found for each mass is listed in Table 4.

Table 3 Relative transverse displacement differences for various masses

Mass ($lbs.$)	Displacement Difference ($in.$)
$\mathbf{2.128}$	0.002
$\mathbf{3.212}$	0.002
$\mathbf{5.134}$	0.0035
$\mathbf{10.96}$	0.0065

Table 4 Shear moduli for various masses

Mass ($lbs.$)	**Angle of Rotation** (deg)	**Shear Modulus,** G ($lb/in.^2$)
$\mathbf{2.128}$	0.1553	5.93×10^6
$\mathbf{3.212}$	0.1553	8.95×10^6
$\mathbf{5.134}$	0.272	8.18×10^6
$\mathbf{10.96}$	0.505	9.40×10^6

Similar to the tensile modulus experiment, the angle of rotation increases as the weight of the mass increases. Unlike the former experiment, however, the derived moduli vary more significantly. The lack of precision could be attributed to the small distance across the width of the beam when taking measurements, the small distance off-center from the beam's centroid to the hanging mass, and general fatigue of the beam after many cycles of bending. Additionally, these results can be only an approximation of the beam's reaction to torque as the hanging mass is not a pure torque since it produces a shear force as well.

3.3 Digital versus Analog Measurements

The digital and analog displacement indicators are compared for the displacement of the beam in the direction of bending along the length of the beam. Listed in Table 5 are the data for the various masses. Figures 9-11 show the plots for each digital-analog pair.

Table 5 Measurements of displacement along the beam for various masses comparing digital to analog displacement indicators

Mass ($lbs.$)	$\mathbf{3\ in.}$	$\mathbf{5\ in.}$	$\mathbf{7\ in.}$	$\mathbf{9\ in.}$	$\mathbf{11\ in.}$	$\mathbf{13\ in.}$
$\mathbf{2.128}$ **(d)**	$0.0085\ in.$	$0.0205\ in.$	$0.0350\ in.$	$0.0515\ in.$	$0.0705\ in.$	$0.0880\ in.$
(a)	$0.009\ in.$	$0.019\ in.$	$0.033\ in.$	$0.051\ in.$	$0.069\ in.$	$0.088\ in.$
$\mathbf{3.212}$ **(d)**	$0.0140\ in.$	$0.0310\ in.$	$0.0450\ in.$	$0.0800\ in.$	$0.1085\ in.$	$0.1385\ in.$
(a)	$0.0014\ in.$	$0.030\ in.$	$0.052\ in.$	$0.080\ in.$	$0.106\ in.$	$0.133\ in.$
$\mathbf{5.134}$ **(d)**	$0.0220\ in.$	$0.0505\ in.$	$0.0880\ in.$	$0.1300\ in.$	$0.1770\ in.$	$0.2265\ in.$
(a)	$0.024\ in.$	$0.050\ in.$	$0.086\ in.$	$0.132\ in.$	$0.174\ in.$	$0.221\ in.$

Observably, the data from the digital and analog displacement indicators are close in value aside from the difference in instrument precision. As the mass is increased, the difference between the digital and analog displacement increases, particularly as the distance from the support becomes further.

4. CONCLUSIONS

A simple beam has been examined to determine the tensile modulus and shear modulus of the beam. Measurements of the beam's deflection along the length of the beam were made to determine the tensile modulus using Euler-Bernoulli beam theory. Similarly, measurements of the relative transverse displacements at the end of the beam were used to determine the shear modulus. In addition, the digital and analog displacement indicators were compared. The following conclusions are formed:

1. The beam has a tensile modulus, or Young's modulus, of $8.50 \times 10^6\ lb/in.^2$, within the uncertainty.
2. The beam has a shear modulus of $8.12 \times 10^6\ lb/in.^2$, within the uncertainty.

3. The digital and analog displacement indicators are mutually precise with the measurements becoming less precise as the weight of the hanging mass is increased and measurements are taken further down the length of the beam.

REFERENCES

Hallauer W. L. Jr. and Devenport W. J., 2006, *AOE 3054 Experimental Methods Course*

APPENDIX: UNCERTAINTY CALCULATIONS

To obtain uncertainties in results R derived from these measurements, uncertainties were combined using the root sum square equation,

$$\delta(R) = \sqrt{\left(\frac{\partial R}{\partial a}\delta(a)\right)^2 + \left(\frac{\partial R}{\partial b}\delta(b)\right)^2 + \left(\frac{\partial R}{\partial c}\delta(c)\right)^2 + \cdots}$$

where a, b, c, … are the measurements on which R depends. The partial derivatives were estimated numerically.

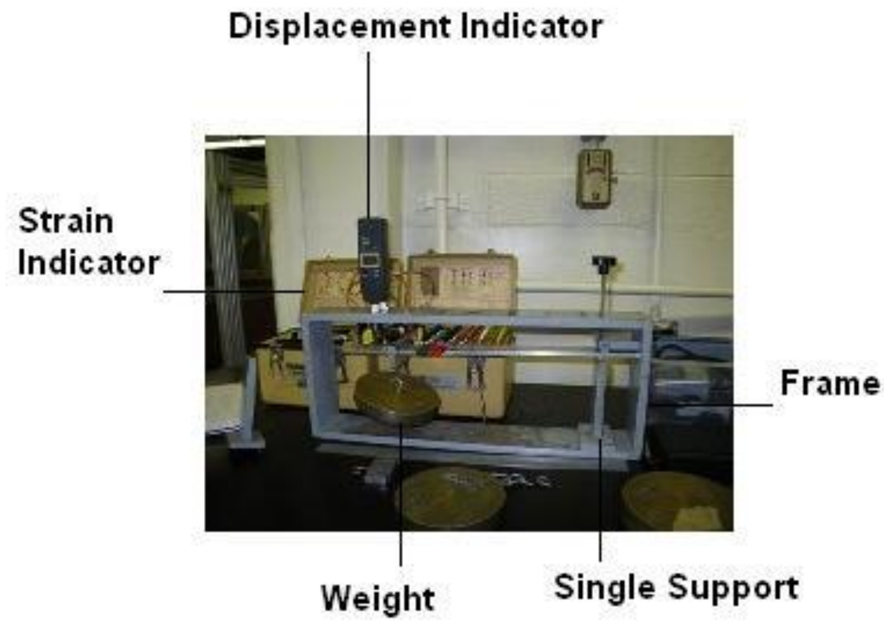

Figure 1. Photograph of the frame, beam, hanging mass, and displacement indicator.

Figure 2. Photograph of the cantilever beam support system.

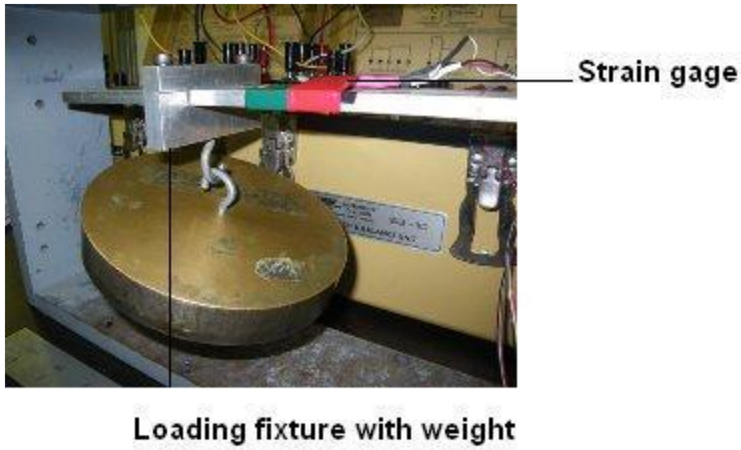

Figure 3. Photograph of the tensile modulus experiment setup.

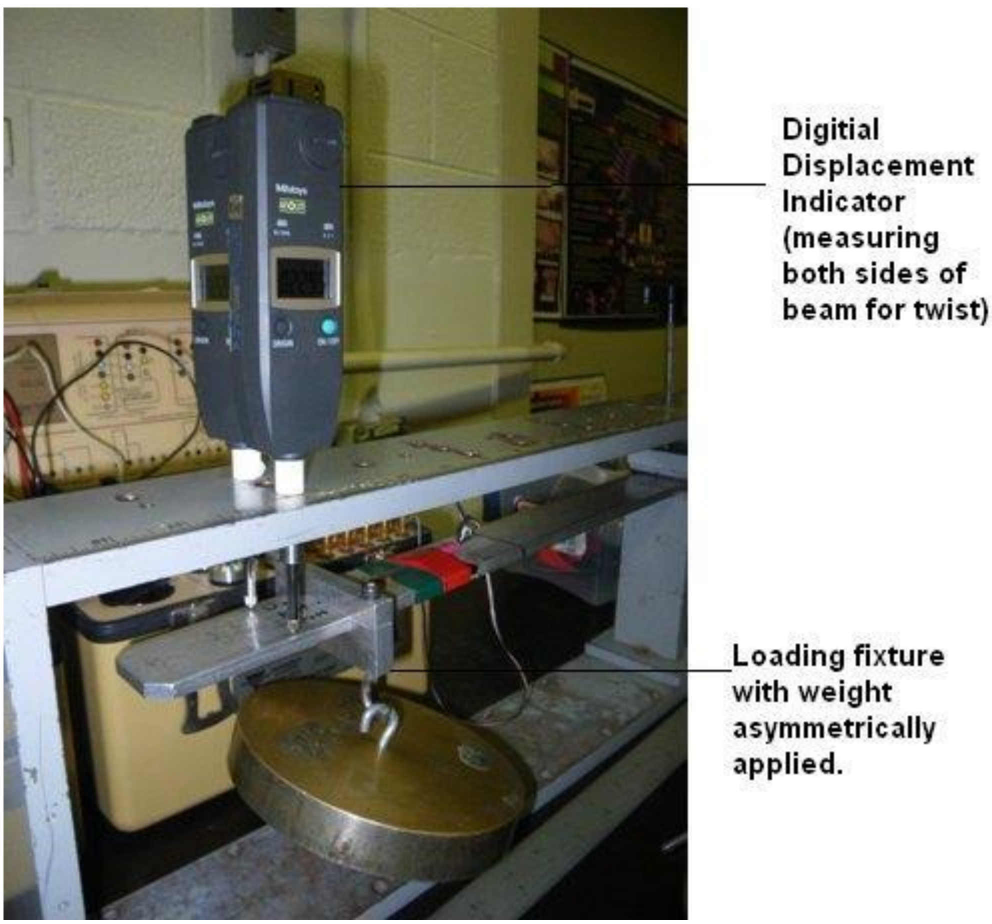

Figure 4. Photograph of the shear modulus experiment setup.

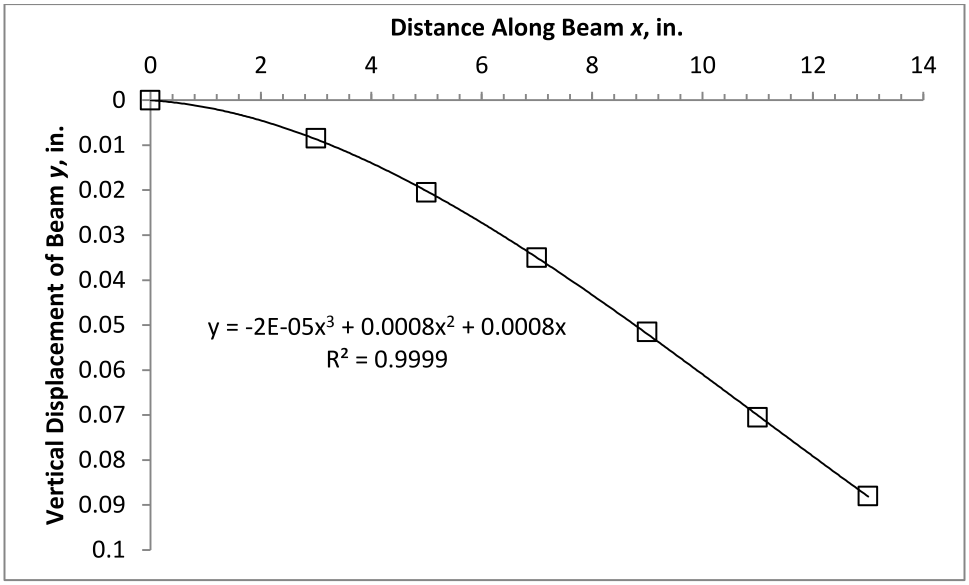

Figure 5. Vertical displacement along beam for hanging mass of $2.128\ lbs$.

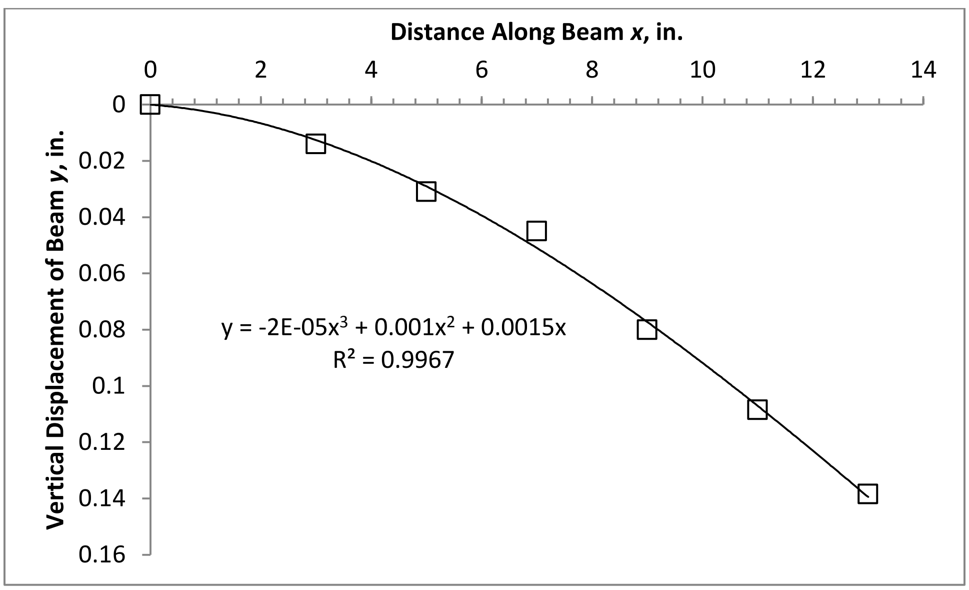

Figure 6. Vertical displacement along beam for hanging mass of $3.212\ lbs$.

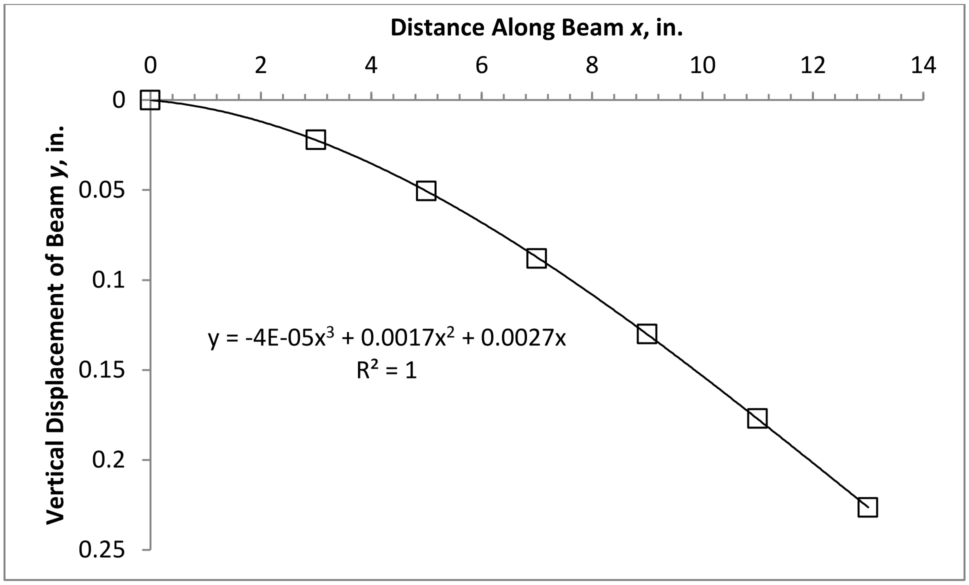

Figure 7. Vertical displacement along beam for hanging mass of $5.134\ lbs$.

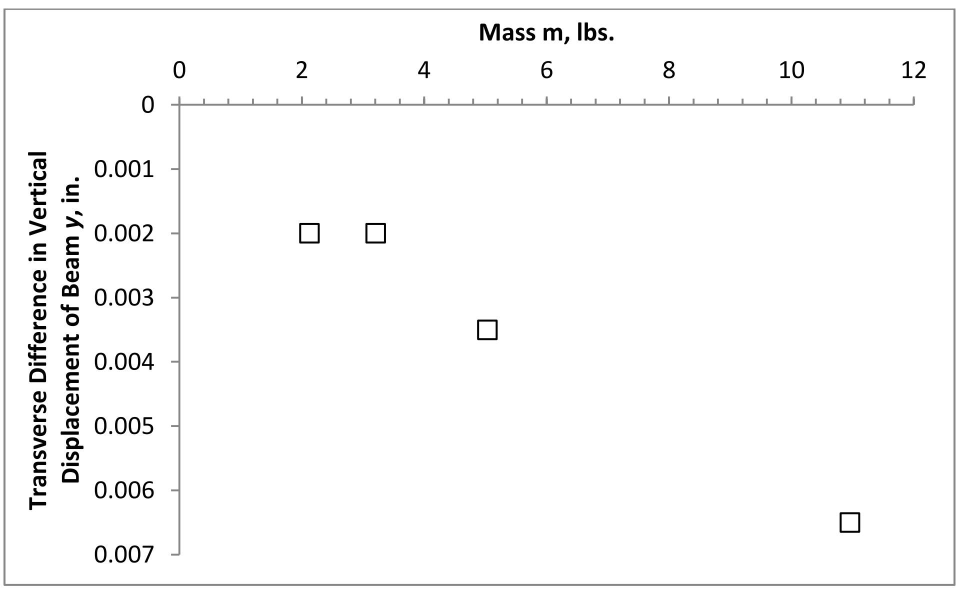

Figure 8. Transverse difference in vertical displacement for varying hanging masses.

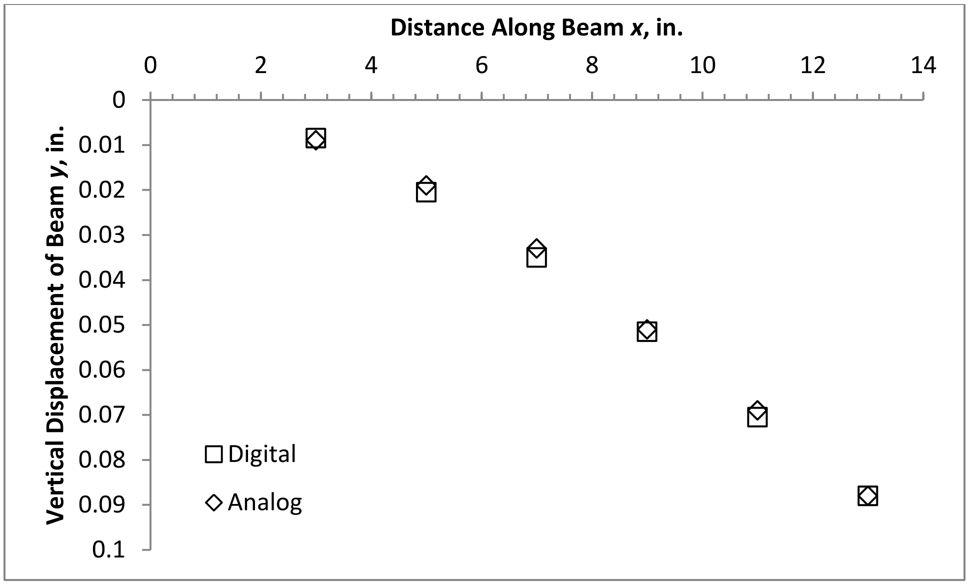

Figure 9. Digital and analog data for vertical displacement for mass of 2.128 lbs.

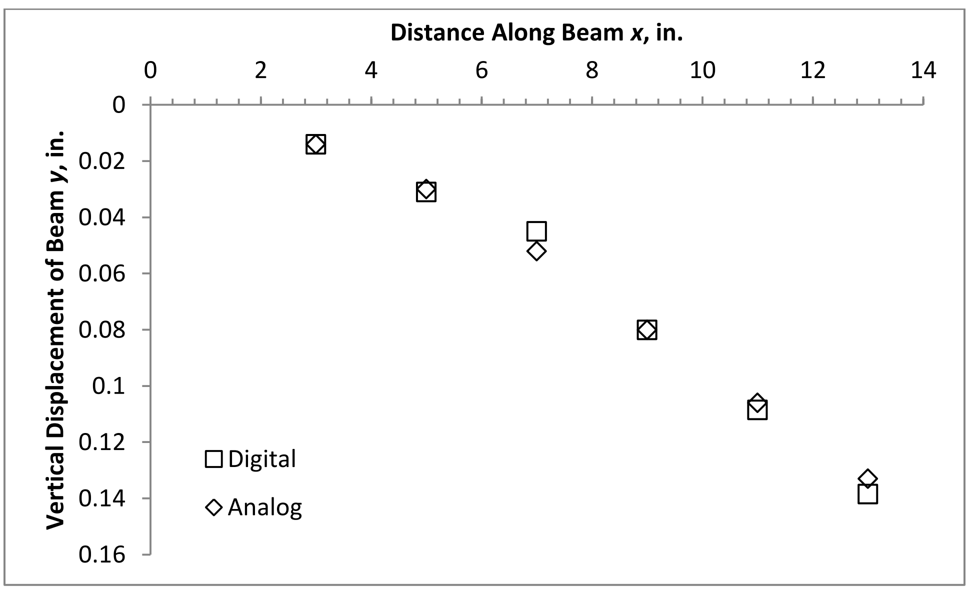

Figure 10. Digital and analog data for vertical displacement for mass of 3.212 lbs.

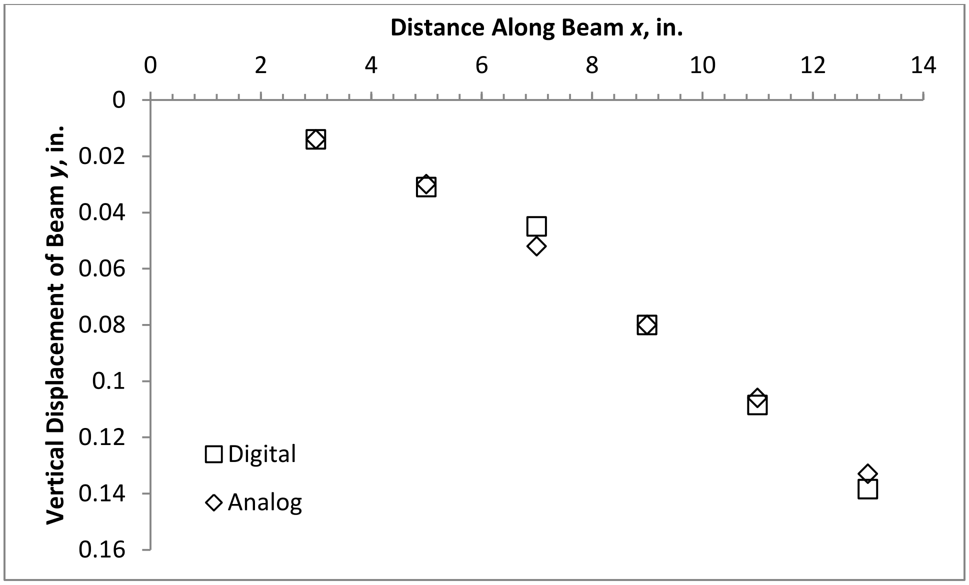

Figure 11. Digital and analog data for vertical displacement for mass of $5.134\ lbs$.